75 years of mathematics in Mexico

The Instituto de Matemáticas of the National Autonomous University of Mexico (UNAM) is the oldest of its kind in Latin America. In 2017 we celebrate our 75th anniversary with a conference that will cover a wide range of today's mathematics and hold top quality talks with insights of some of the main trends in mathematics in the years to come.

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Venue

- The conference will take place at the Auditorio Alberto Barajas Celis of the Facultad de Ciencias-UNAM (morning session) and the Auditorio Alfonso Nápoles Gándara of the Instituto de Matemáticas-UNAM (after lunch session). The web page of the conference is

  `https://www.75years-im.matem.unam.mx`.

- Lunch will be served (only by invitation) every day at 13:15 at the terrace of the new building of the Instituto de Matemáticas (see maps below).
- On Sunday 3/12 there will be a Welcome Cocktail at the Hotel Radisson starting at 18:00.

- On Monday 4/12 there will be an Inauguration Cocktail at the Auditorio Nápoles Gándara of the Instituto de Matemáticas starting at 17:00.
- On Thursday 7/12 there will be the inauguration of an exhibition called "Imaginario Matemático", starting at 18:00, celebrating the first 25 year of Universum Museum. There will be a private view of the exhibition, together with drinks and snacks offered by our Institute and the French Embassy.

Invited speakers

- Noga Alon (Tel Aviv University, Israel)
- Luis Caffarelli (University of Texas at Austin, USA)
- Mónica Clapp (Instituto de Matemáticas-UNAM, Mexico)
- José Antonio de la Peña (Instituto de Matemáticas-UNAM, Mexico)
- David Gabai (Princeton University, USA)
- Christof Geiss (Instituto de Matemáticas-UNAM, Mexico)
- Adolfo Guillot (Instituto de Matemáticas-UNAM, Mexico)
- Mike Hopkins (University of Harvard, USA)
- Isabel Hubard (Instituto de Matemáticas-UNAM, Mexico)
- June Huh (Princeton University, USA)
- Kristin Lauter (Microsoft Research, USA)
- Pierre Louis Lions (College de France, France)
- Dusa McDuff (Barnard College, Columbia University, USA)
- Sylvie Méléard (École Polytechnique, France)
- John Milnor (University of New York at Stonybrook, USA)
- Shigefumi Mori (RIMS, Kyoto, Japan)
- Nizar Touzi (École Polytechnique, France)
- Gerónimo Uribe (Instituto de Matemáticas-UNAM, Mexico)
- Alberto Verjovsky (Instituto de Matemáticas-UNAM, Mexico)

Distinguished guests

- Nassif Ghoussoub (University of British Columbia)
- Jacob Palis (IMPA, Brazil)

Internet Access

At the Auditorio Alberto Barajas Celis:
Name: `75years`
Password: `imate75`

At the Instituto de Matemáticas:
Name: `75Aniversario-IM`
Password: `Bg6&3697h8`

Sponsors

Abstracts

Title: Structure, Randomness and Universality in Graph Theory.

Noga Alon
Tel Aviv University, Israel

Abstract: What is the minimum possible number of vertices of a graph that contains every k-vertex graph as an induced subgraph? What is the minimum possible number of edges in a graph that contains every k-vertex graph with maximum degree 3 as a subgraph? These questions and related ones were initiated by Rado in the 60s, and received a considerable amount of attention over the years, partly motivated by algorithmic applications. The study of the subject combines probabilistic arguments and explicit, structured constructions. I will survey the topic focusing on a recent asymptotic solution of the first question, where an asymptotic formula, improving earlier estimates by several researchers, is obtained by combining combinatorial and probabilistic arguments with group theoretic tools.

Title: Diverse models on segregation.

Luis Caffarelli
University of Texas at Austin, USA

Abstract: In this lecture we will give an overview of the analytical properties of several models describing segregation patterns under different circumstances depending on the interaction process, adjacent processes, like particle annihilation, segregation at a distance, and the interaction of species that diffuse continuously as insects with those dispersed by wind like some seeds.

Title: New blow up profiles for Yamabe type problems.

Mónica Clapp
Instituto de Matemáticas-UNAM, Mexico

Abstract: Many problems in differential geometry are expressed in terms of an elliptic partial differential equation which is conformally invariant. Typical examples are the Yamabe problem or the prescribed scalar curvature problem. The invariance of these equations under dilations gives rise to blow-up phenomena, which makes them hard to solve. It is, thus, important to understand these phenomena, in other words, to obtain information on energy level, the location and the limit profile of the blow-up. A particular profile has been profusely studied: that given by the so-called standard bubble, i.e., the solution to the Yamabe problem on the round sphere. It has been successfully used

to construct solutions of many different types of problems. In this talk we will exhibit other blow-up profiles, which arise by considering special types of symmetries, and we will use them to produce solutions of some elliptic PDEs.

Title: The 4-Dimensional Light Bulb Theorem.

David Gabai
Princeton University, USA

Abstract: We generalize the classical light bulb theorem to four dimensions, i.e. a smooth 2-sphere in $S^2 \times S^2$ that intersects $S^2 \times 0$ once and is homologous to $0 \times S^2$ is smoothly isotopically standard.

Title: Quivers, Symmetrizable Cartan Matrices and Representation Theory.

Christof Geiss
Instituto de Matemáticas-UNAM, Mexico

Abstract: We briefly review the representation theory of complex semisimple Lie algebras, including Lusztig's semicanonical basis for the symmetric cases. We note that the basic theory of semismiple Lie algebras is completely uniform for all Dynkin types. However, some more advanced geometric constructions, like the semicanonical basis work well only for the symmetric cases due to their close link to quiver representations. The same phenomenon occurs for Kac-Moody Lie algebras.

In joint work with B. Leclerc and J. Schröer we proposed a 1-Iwanaga Gorenstein algebra H, defined over an arbitrary field K, associated to the datum of a symmetrizable Cartan Matrix C, a symmetrizer D of C and an orientation Ω. The H-modules of finite projective dimension behave in many aspects like the modules over a hereditary algebra, and we can associate to H a kind of preprojective algebra Π.

If we look, for K algebraically closed, at the varieties of representations of Π which admit a filtration by generalized simples, we find that the components of maximal dimension provide a realization of the crystal $B(-\infty)$ corresponding to C. For K being the complex numbers we can construct, following ideas of Lusztig, an algebra of constructible functions which contains a family of "semicanonical functions". Those are naturally parametrized by the above mentioned components of maximal dimension. Modulo a conjecture about the support of the functions in the "Serre ideal" the semicanonical functions yield a basis of the enveloping algebra U(n) of the positive part of the Kac-Moody Lie algebra g(C).

Title: Single-valued solutions of complex differential equations.

Adolfo Guillot
Instituto de Matemáticas-UNAM, Mexico

Abstract: The success of the theory of elliptic functions in the nineteenth century motivated the quest for other special functions which were solutions of algebraic differential equations in the complex domain. On its turn, this led to the problem of understanding those differential equations that do not have multivalued solutions. We will talk about some instances of this problem for the autonomous differential equations given by holomorphic vector fields on complex manifolds, and particularly of some results in the case of surfaces.

Title: Homotopy theory and algebraic vector bundles.

Mike Hopkins
University of Harvard, USA

Abstract: This talk will describe joint work with Aravind Asok and Jean Fasel using the methods of homotopy theory to construct new examples of algebraic vector bundles. I will describe a natural conjecture which, if true, implies that over the complex numbers the classification of algebraic vector bundles over smooth affine varieties admitting an algebraic cell decomposition coincides with the classification of topological complex vector bundles.

Title: Twisted honeycombs revisited.

Isabel Hubard
Instituto de Matemáticas-UNAM, Mexico

Abstract: In the 70's Coxeter considered the 4-dimensional regular convex polytopes and used the so-called Petrie Polygons to obtain quotients of the polytopes that while having all possible rotational symmetry, lack of reflectional symmetry. He called this objects Twisted Honeycombs. Nowadays, objects with such symmetry properties are often called chiral. In this talk I will review Coxeter's twisted honeycombs and will connect them to chiral maniplexes and chiral polytopes of full rank (objects that I will define). We will also see a natural way to extend Coxeter's work.

Title: Hard Lefschetz and Hodge-Riemann for finite projective geometries.

June Huh
Princeton University, USA

Abstract: From any finite projective space, I will construct a ring that satisfies all the known properties of the cohomology ring of a smooth projective variety. I will indicate proofs of the hard Lefschetz theorem and the Hodge-Riemann relation in this context, and ask whether the ring is the cohomology ring of a geometric object.

Title: Supersingular Isogeny Graphs in Cryptography.

Kristin Lauter
Microsoft Research, USA

Abstract: Supersingular Isogeny Graphs were proposed for use in Cryptography in 2006 by Charles-Goren-Lauter, and are currently being considered as candidates for standardization in several tracks of the 2017 NIST Post-Quantum Cryptography International Competition. These are Ramanujan graphs whose nodes are supersingular elliptic curves and edges are isogenies between them. This talk will introduce the hard problems and cryptographic applications in this space, and discuss a surprising connection to quantum arithmetic.

Title: TBA.

Pierre Louis Lions
College de France, France

Abstract: TBA.

Title: Symplectic Topology Today.

Dusa McDuff
Barnard College, Columbia University, USA

Abstract: This will be an introduction to the world of symplectic topology, explaining some of its roots, some of the major recent advances, and some currently open questions.

Title: Stochastic dynamics for adaptation and evolution of microorganisms.

Sylvie Méléard
École Polytechnique, France

Abstract: Understanding the adaptation and evolution of populations is a huge challenge, in particular for microorganisms since it plays a main role in the virulence evolution or in bacterial antibiotics resistances. We propose a general eco-evolutionary stochastic model of population dynamics with clonal reproduction and mutations, including competition for resources and exchange of genes. We study some asymptotics of this general birth and death process depending on the respective demographic, ecological and exchange time scales. We show how the gene exchanges can drastically affect the evolutionary outcomes.

Title: Two Moduli Spaces.

John Milnor
University of New York at Stonybrook, USA

Abstract: A discussion of two moduli spaces and their awkward topologies: first the space of divisors on the Riemann sphere modulo the action of Moebius automophisms; and second the (compactified) space of curves in the complex projective plane modulo projective automorphisms.This is joint work with Araceli Bonifant.

Title: On the classification of algebraic varieties.

Shigefumi Mori
RIMS, Kyoto, Japan

Abstract: In my talk I will present my personal views on the area around my research; I have been studying algebraic varieties through rational curves on them. I was first interested in a special problem called the Hartshorne Conjecture, and when I solved it I encountered a notion called an extremal ray as a biproduct, through which I got attracted to the biregular classification and the minimal model program, and furthermore to a general theory of higher dimensional birational classification. I will present them to a wider audience including algebraic geometers.

Title: Degenerations of algebras and modules.

José Antonio de la Peña
Instituto de Matemáticas-UNAM, Mexico

Abstract: Let A be a finite dimensional k-algebra. We introduce some concepts on the geometry of representations and its applications.

1. The representation type of the algebra (finite, tame or wild) can be read in the dimension of the module variety $\mathrm{mod}_A(d)$ and the orbits $G(d)X$, where $G(d)$ if the group determining the iso-classes in $\mathrm{mod}_A(d)$, $d \in \mathbb{N}$.

2. Given two algebras A and B, we say that B is a degeneration of A if there is an algebraic family $(A_z)_z \in Z$ such that A_z is isomorphic to A in a open and dense subset of Z and A_{z_0} is isomorphic to B in some $z_0 \in Z$. If B is a degeneration of A then $\dim_k H^n(B) \geq \dim_k H^n(A)$, where H^n denotes the Hochschild cohomology.

3. Geiss degeneration theorem: a degeneration of a wild algebra is wild.

4. Denote q_A the (quadratic) Tits form of A, then:

   a) If A is representation finite then q_A is weakly positive (i.e. $q_A(u) > 0$ for $0 \neq u$ any vector with $0 \leq$ entries).

   b) If A is tame the q_A is weakly non-negative (ie. $q_A(u) \geq 0$ for any vector u with $0 \leq$ entries).

5. Bruestle-JAP-Skowronski theorem: let A be a strongly simply connected algebra then A is tame if and only if q_A is wnn.

6. We show that A is strongly simply connected if and only if the trace of the Coxeter matrix satisfies $\mathrm{tr}(\Phi_A) = -1$.

Title: Second order backward SDEs, and application to stochastic differential games.

Nizar Touzi
École Polytechnique, France

Abstract: Backward SDEs can be viewed as the path-dependent analogue of Sobolev solutions of parabolic second order PDEs. We provide some recent results in the context of random horizon. This corresponds to an extension of elliptic PDEs. As an application of these results, we provide a systematic method for solving general Principal-Agent problems with possibly infinite horizon. Our main result reduces such Stackelberg stochastic differential games to a standard stochastic control problem, which may be addressed by the standard tools of control theory.

Title: On random trees and Lévy type processes.

Gerónimo Uribe
Instituto de Matemáticas-UNAM, Mexico

Abstract: The year 1875 marks the introduction of stochastic elements in the modeling of population growth with Galton and Watson's landmark paper On the Probability of the Extinction of Families. Fast-forward to 1991 when the Galton-Watson model was reinterpreted as one of random genealogical trees by Aldous. This enabled the construction of a universal scaling limit of discrete Galton-Watson trees via a random metric space with tree-like structure called the Continuum Random Tree. In this talk, we outline some aspects of random models of discrete trees and of their proven or conjectured scaling limits. Main aims will be the introduction of time in the models in order to distinguish chronology from genealogy and a sample of locally compact random real trees. Lévy processes will play a prominent role.

Title: Compact 3-dimensional geometric solenoidal manifolds.

Alberto Verjovsky
Instituto de Matemáticas-UNAM, Mexico

Abstract: We present several results about solenoidal manifolds motivated by results by Dennis Sullivan in [1] with commentaries developed in [2] and on a join project with Dennis Sullivan [3]. Solenoidal manifolds of dimension n are topological spaces which are locally homeomorphic to the product of a Cantor set with an open subset of $\mathbb{R}^n$. Geometric 3-dimensional solenoidal manifolds are the analog of geometric 3-manifolds in the sense of Thurston. We will give some results related to 3-dimensional geometric solenoidal manifolds.

[1] D. Sullivan, *Solenoidal manifolds*, J. Singul. **9** (2014), 203–205.
[2] A. Verjovsky, *Commentaries on the paper "Solenoidal manifolds" by Dennis Sullivan*, J. Singul. **9** (2014), 245–251.
[3] D. Sullivan, A. Verjovsky, *Compact 3-dimensional geometric solenoidal manifolds*. In preparation.